

15. (a) Find asymptotic estimates of a fundamental set of solutions of the difference equation

$$y(n+2) + \frac{(n+1)}{(n+2)}y(n+1) - \frac{2n}{n+2}y(n) = 0 \text{ using}$$

Poincare - person theorem.

Or

- (b) State and prove Poincare's theorem.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Abel's lemma.
17. Solve the system $y(n+1) = Ay(n) + g(n)$, where
 $A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$, $y(n) = \begin{pmatrix} n \\ 1 \end{pmatrix}$, $y(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.
18. Solve the difference equation
 $x(n+3) - x(n+2) + 2x(n) = 0$, $x(0) = 1$, $x(1) = 1$
 using Z-transform.
19. State and prove Sturm separation theorem.
20. Derive variation of constant formula in asymptotic theory.

APRIL/MAY 2025

23PEMA25D — DIFFERENCE EQUATIONS

Time : Three hours

Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

- When we say the functions $f_1(n), f_2(n), \dots, f_r(n)$ are linearly independent?
- Define the term annihilator.
- State Cayley - Hamilton theorem.
- Consider the system $x(n+1) = A(n)x(n)$ with
 $A(n) = \begin{pmatrix} 0 & (-1)^n \\ (-1)^n & 0 \end{pmatrix}$. Find its Floquet multipliers.
- Find the Z-transform of the sequence $\{a^n\}$ for a fixed real number a , and its region of convergence.
- Define Euclidean norm.



7. Find the limit superior and limit inferior for the following sequence $S: \frac{3}{2}, -\frac{1}{2}, \frac{4}{3}, -\frac{1}{3}, \frac{5}{4}, -\frac{1}{4}, \dots$
8. Show that every solution of the difference equation $\Delta \left(n \Delta x(n-1) - \frac{1}{n} x(n) \right) = 0$ is nonoscillatory.
9. Show that $t^2 \log t + t^3 = O(t^4)$, $t \rightarrow \infty$.
10. State Benzaid – Lutz theorem.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Solve the difference equation $(E-3)(E+2)y(n) = 5(3^n)$.

Or

- (b) Prove that the conditions $1 + p_1 + p_2 > 0$, $1 - p_1 + p_2 > 0$, $1 - p_2 > 0$ are necessary and sufficient for the equilibrium solution of equations $y(n+2) + p_1 y(n+1) + p_2 y(n) = 0$ and $y(n+2) + p_1 y(n+1) + p_2 y(n) = M$ to be asymptotically stable.

12. (a) Find the solution of the difference system

$$x(n+1) = Ax(n), \text{ where } A = \begin{pmatrix} 4 & 1 & 2 \\ 0 & 2 & -4 \\ 0 & 1 & 6 \end{pmatrix}.$$

Or

- (b) Prove that a $k \times k$ matrix is diagonalizable if and only if it has k linearly independent eigenvectors.
13. (a) Obtain the inverse Z -transform of $\tilde{x}(z) = \frac{z(z+1)}{(z-1)^2}$ using power series method.

Or

- (b) Prove that if $x(n) \in l_1$ then
- (i) $\tilde{x}(z)$ is an analytic function for $|z| \geq 1$
- (ii) $|\tilde{x}(z)| \geq \|x\|$ for $|z| \geq 1$
14. (a) Suppose that $p(n) \geq 0$ and $\sup_{n \geq 0} p(n) < \frac{k^k}{k+1^{k+1}}$ then prove that the equation $x(n+1) - x(n) + p(n)x(n-k) = 0$ has a nonoscillatory solution.
- Or
- (b) Prove that every solution of the difference equation $y(n+1) + y(n-1) = \left(2 + \frac{1}{2}(-1)^n\right)y(n)$ is oscillatory.

